CS 225

Data Structures

April 5 – Disjoint Sets Finale + Graphs

Wade Fagen-Ulmschneider, Craig Zilles

Disjoint Sets Find

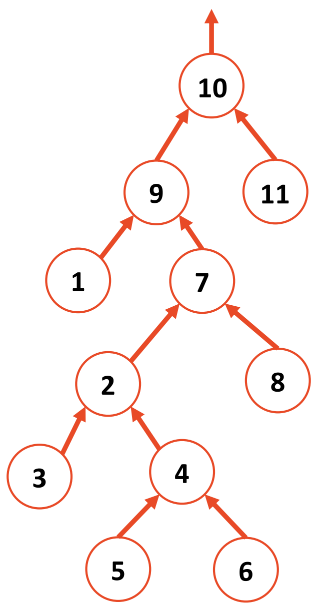
```
int DisjointSets::find(int i) {
  if ( arr_[i] < 0 ) { return i; }
  else { return _find( arr_[i] ); }
}
```

```
void DisjointSets::unionBySize(int root1, int root2) {
  int newSize = arr_[root1] + arr_[root2];

  // If arr_[root1] is less than (more negative), it is the larger set;
  // we union the smaller set, root2, with root1.
  if ( arr_[root1] < arr_[root2] ) {
    arr_[root2] = root1;
    arr_[root1] = newSize;
  }

  // Otherwise, do the opposite:
  else {
    arr_[root1] = root2;
    arr_[root2] = newSize;
  }
}
```

Path Compression



Disjoint Sets Analysis

The **iterated log** function:

The number of times you can take a log of a number.

log*(n) =

0 , $n \leq 1$

1 + log*(log(n)) , $n > 1$

What is **lg*(2^{65536})**?

Disjoint Sets Analysis

In an Disjoint Sets implemented with smart **unions** and path compression on **find**:

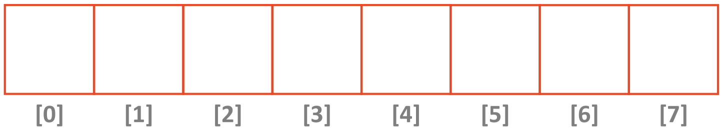
Any sequence of **m** **union** and **find** operations result in the worse case running time of O(____________),

where **n** is the number of items in the Disjoint Sets.

In Review: Data Structures

Array
- Sorted Array
- Unsorted Array
  - Stacks
  - Queues
  - Hashing
  - Heaps
    - Priority Queues
  - UpTrees
    - Disjoint Sets

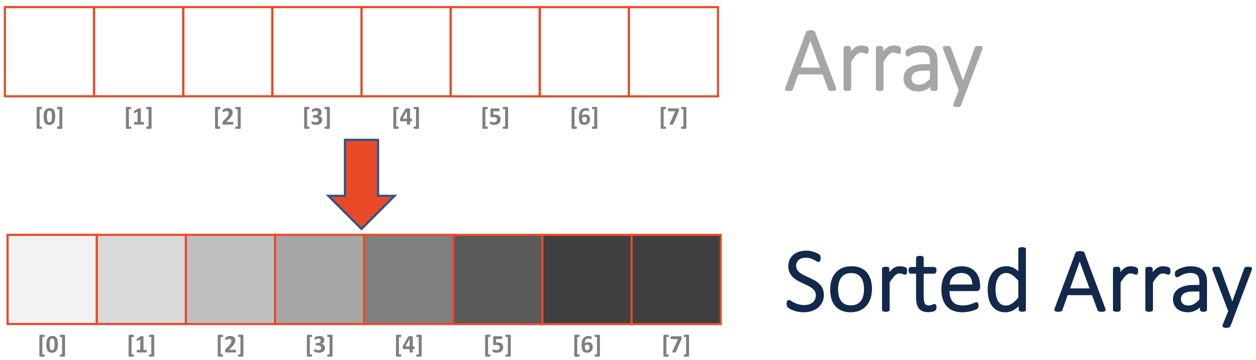
List
- Singly Linked List
- Doubly Linked List
- Trees
  - BTree
  - Binary Tree
    - Huffman Encoding
    - kd-Tree
    - AVL Tree



Array

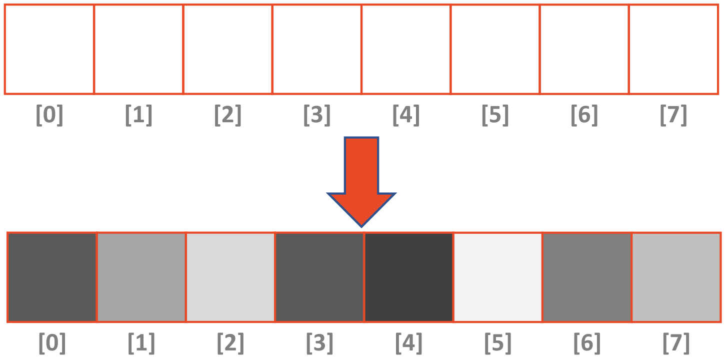
- Constant time access to any element, given an index
  a[k] is accessed in O(1) time, no matter how large the array grows

- Cache-optimized
  Many modern systems cache or pre-fetch nearby memory values due the “Principle of Locality”. Therefore, arrays often perform faster than lists in identical operations.



- Efficient general search structure
  Searches on the sort property run in O(lg(n)) with Binary Search

- Inefficient insert/remove
  Elements must be inserted and removed at the location dictated by the sort property, resulting shifting the array in memory – an O(n) operation

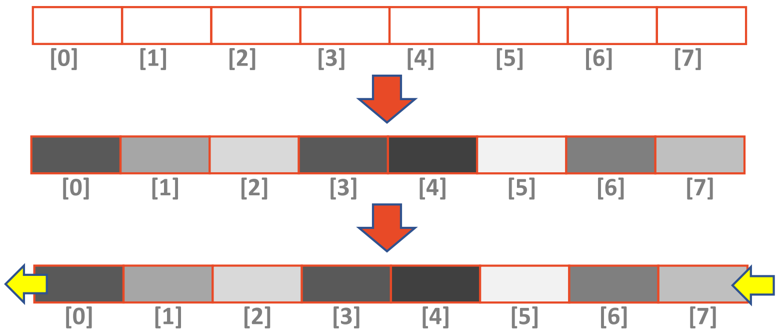


Array

Unsorted Array

- Constant time add/remove at the beginning/end
  Amortized O(1) insert and remove from the front and of the array
  <u>Idea:</u> Double on resize

- Inefficient global search structure
  With no sort property, all searches must iterate the entire array; O(n) time



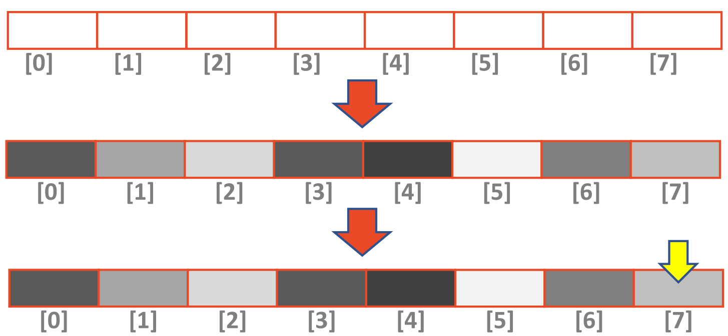
Array

Unsorted Array

Queue (FIFO)

- First In First Out (FIFO) ordering of data
  Maintains an arrival ordering of tasks, jobs, or data

- All ADT operations are constant time operations
  enqueue() and dequeue() both run in O(1) time



Array

Unsorted Array

Stack (LIFO)

- Last In First Out (LIFO) ordering of data
  Maintains a "most recently added" list of data

- All ADT operations are constant time operations
  push() and pop() both run in O(1) time

In Review: Data Structures

Array
- Sorted Array
- Unsorted Array
  - Stacks
  - Queues
  - Hashing
  - Heaps
    - Priority Queues
  - UpTrees
    - Disjoint Sets

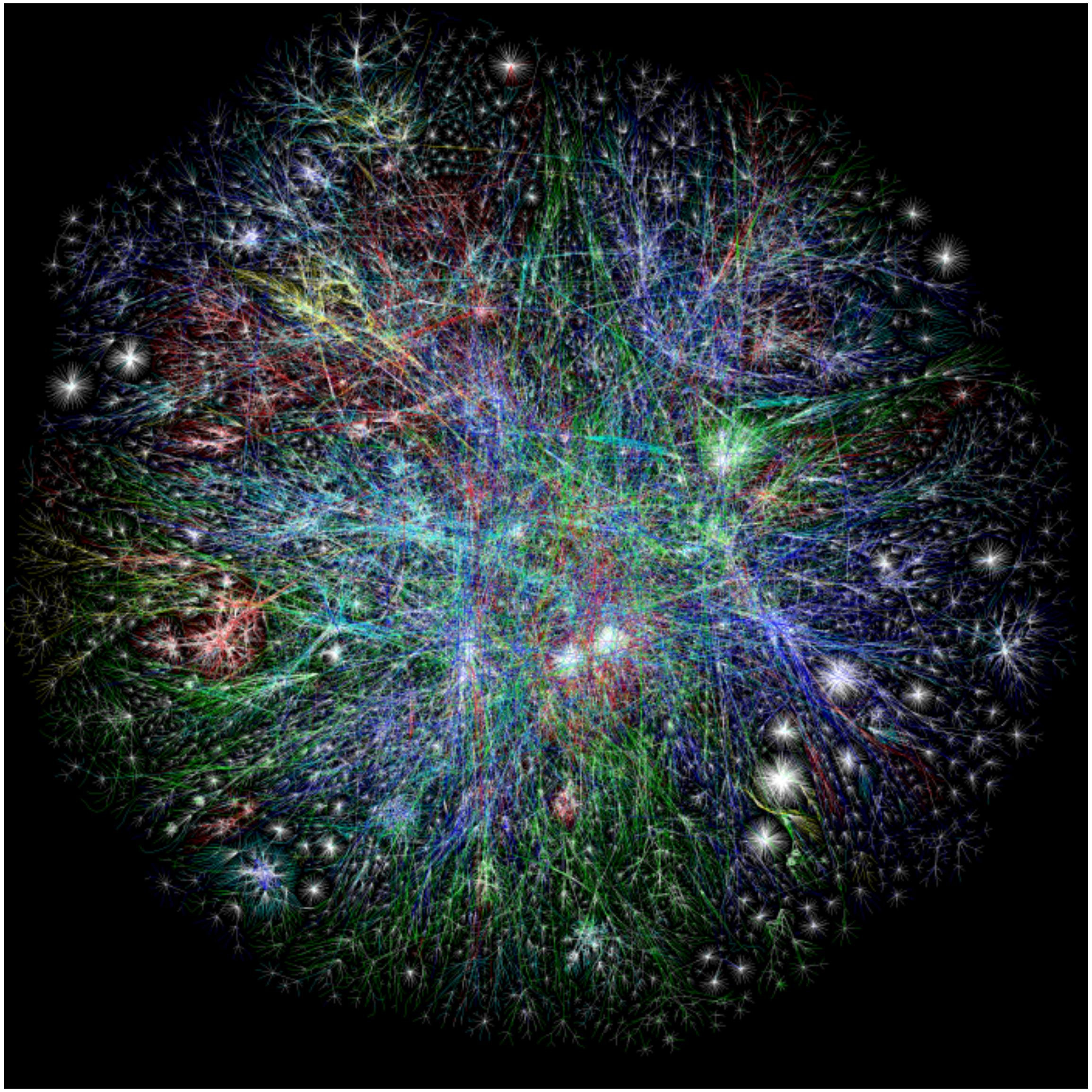
List
- Doubly Linked List
- Trees
  - BTree
  - Binary Tree
    - Huffman Encoding
    - kd-Tree
    - AVL Tree

In Review: Data Structures

Array
- Sorted Array
- Unsorted Array
  - Stacks
  - Queues
  - Hashing
  - Heaps
    - Priority Queues
  - UpTrees
    - Disjoint Sets

Graphs

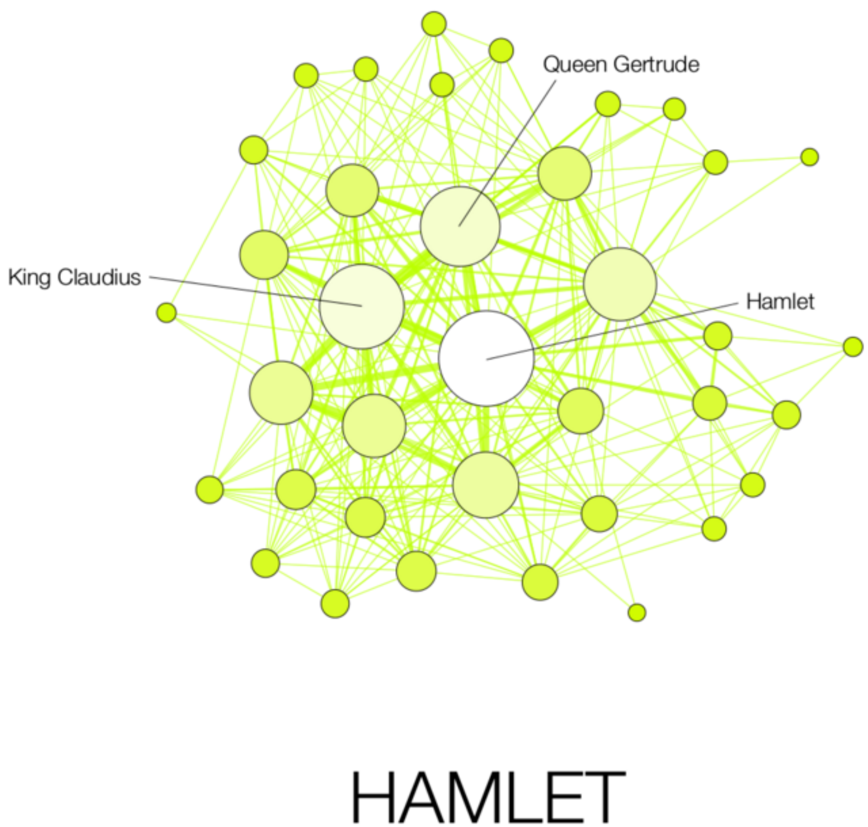
List
- Doubly Linked List
- Skip List
- Trees
  - BTree
  - Binary Tree
    - Huffman Encoding
    - kd-Tree
    - AVL Tree

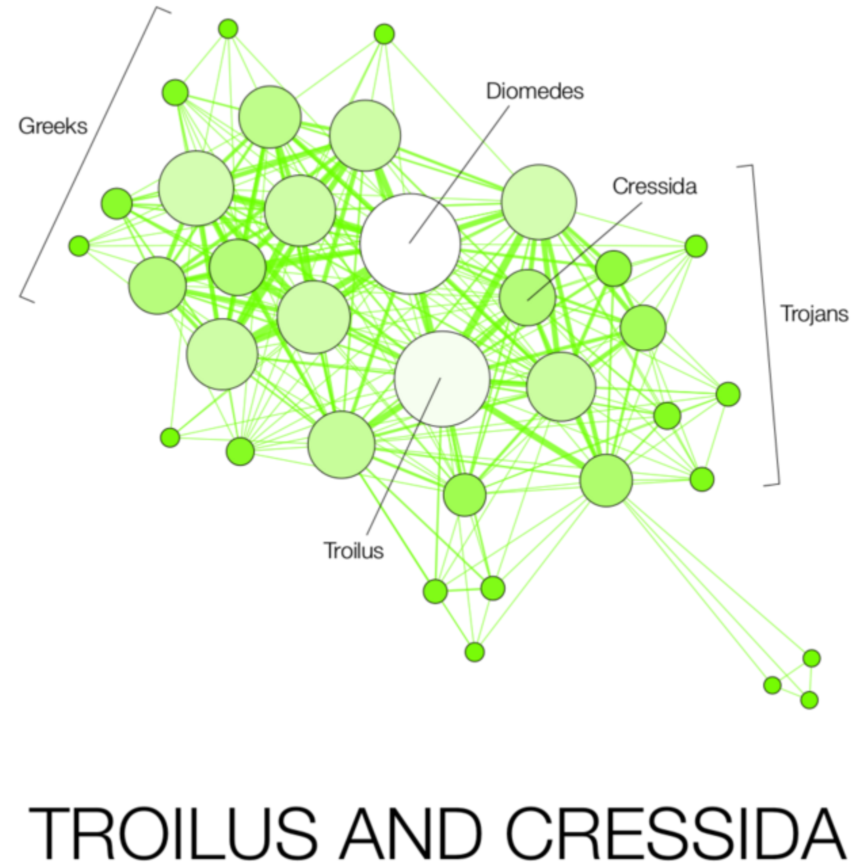


The Internet 2003

The OPTE Project (2003)

Map of the entire internet; nodes are routers; edges are connections.

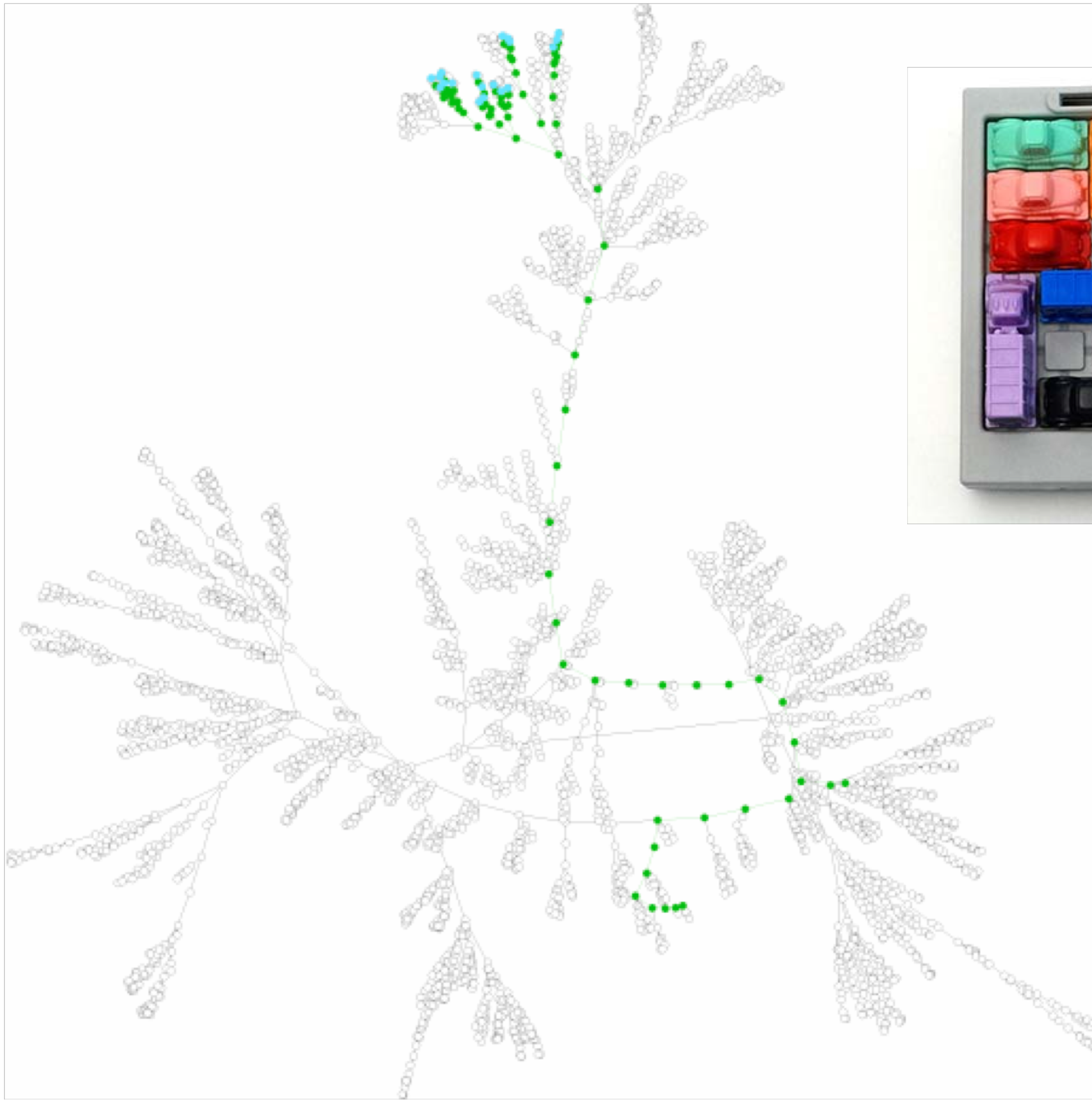




Who's the real main character in Shakespearean tragedies?

Martin Grandjean (2016)

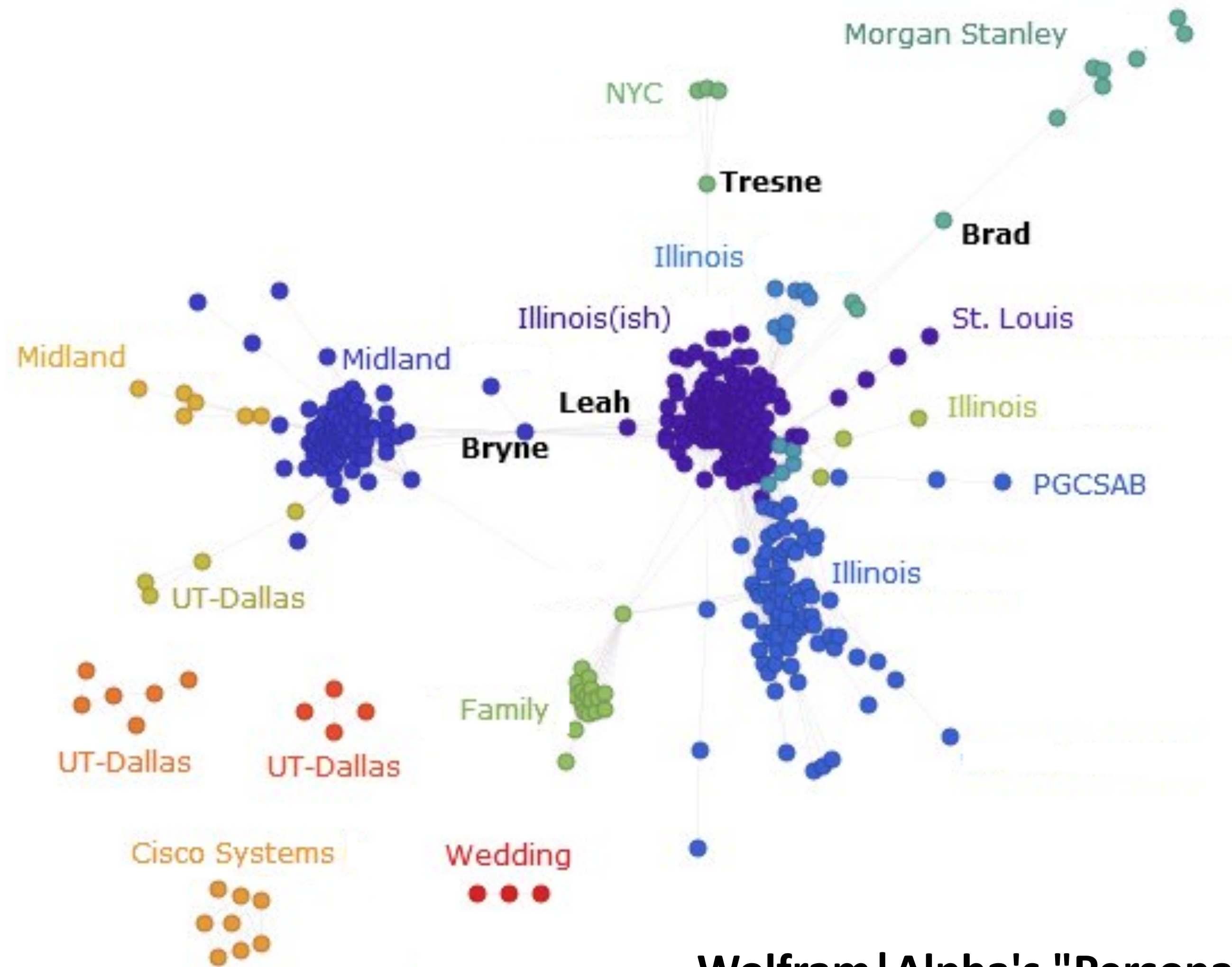
https://www.pbs.org/newshour/arts/whos-the-real-main-character-in-shakespearen-tragedies-heres-what-the-data-say



“Rush Hour” Solution

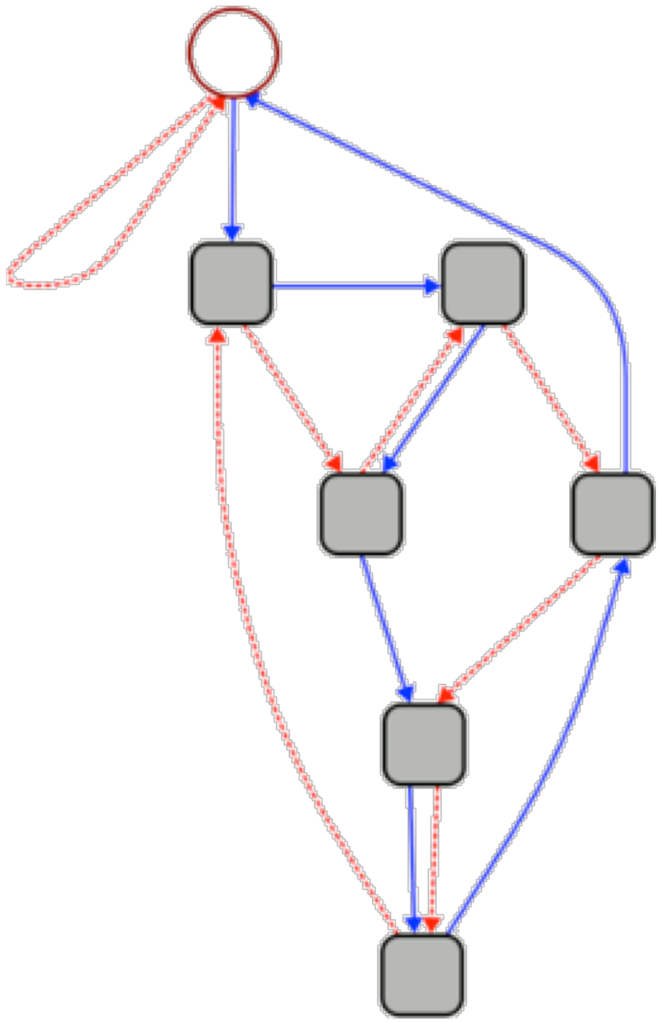
Unknown Source

Presented by Cinda Heeren, 2016



Wolfram|Alpha's "Personal Analytics“ for Facebook

Generated: April 2013 using Wade Fagen-Ulmschneider’s Profile Data

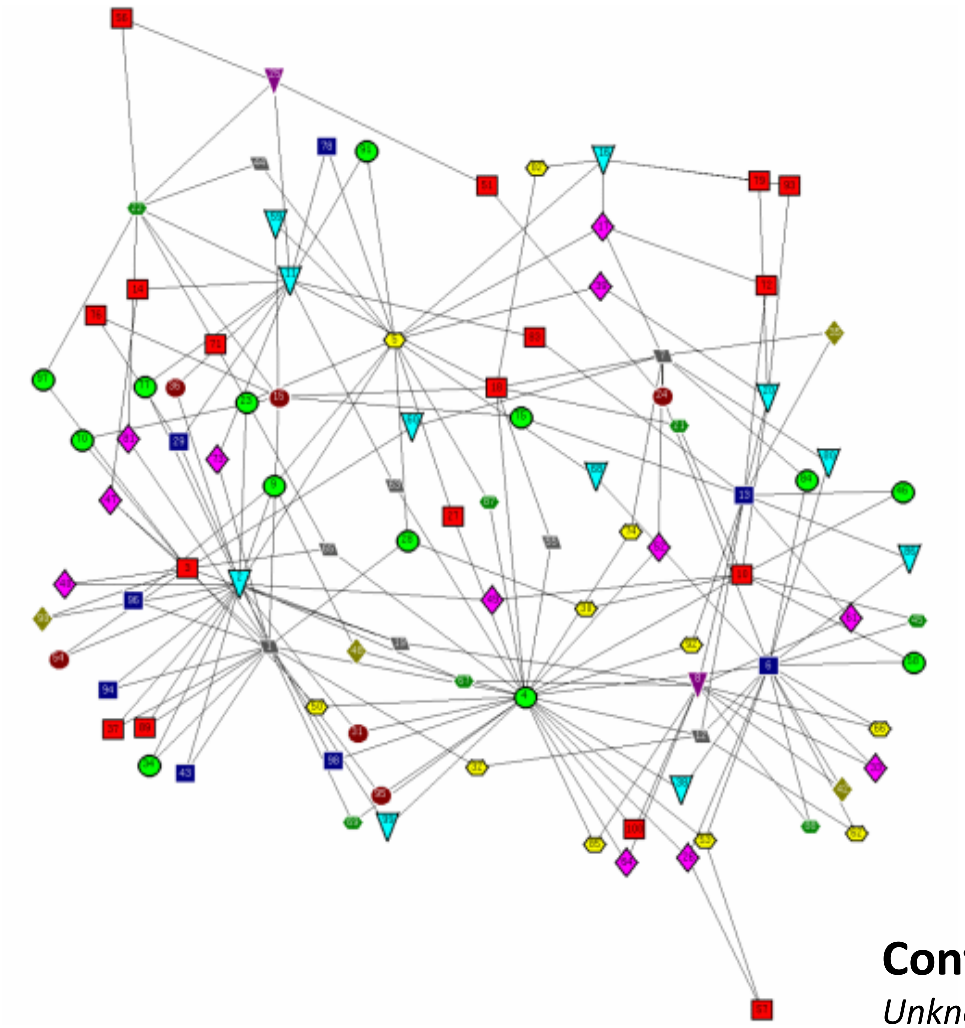


This graph can be used to quickly calculate whether a given number is divisible by 7.

1. Start at the circle node at the top.
2. For each digit **d** in the given number, follow **d** blue (solid) edges in succession. As you move from one digit to the next, follow **1** red (dashed) edge.
3. If you end up back at the circle node, your number is divisible by 7.

3703

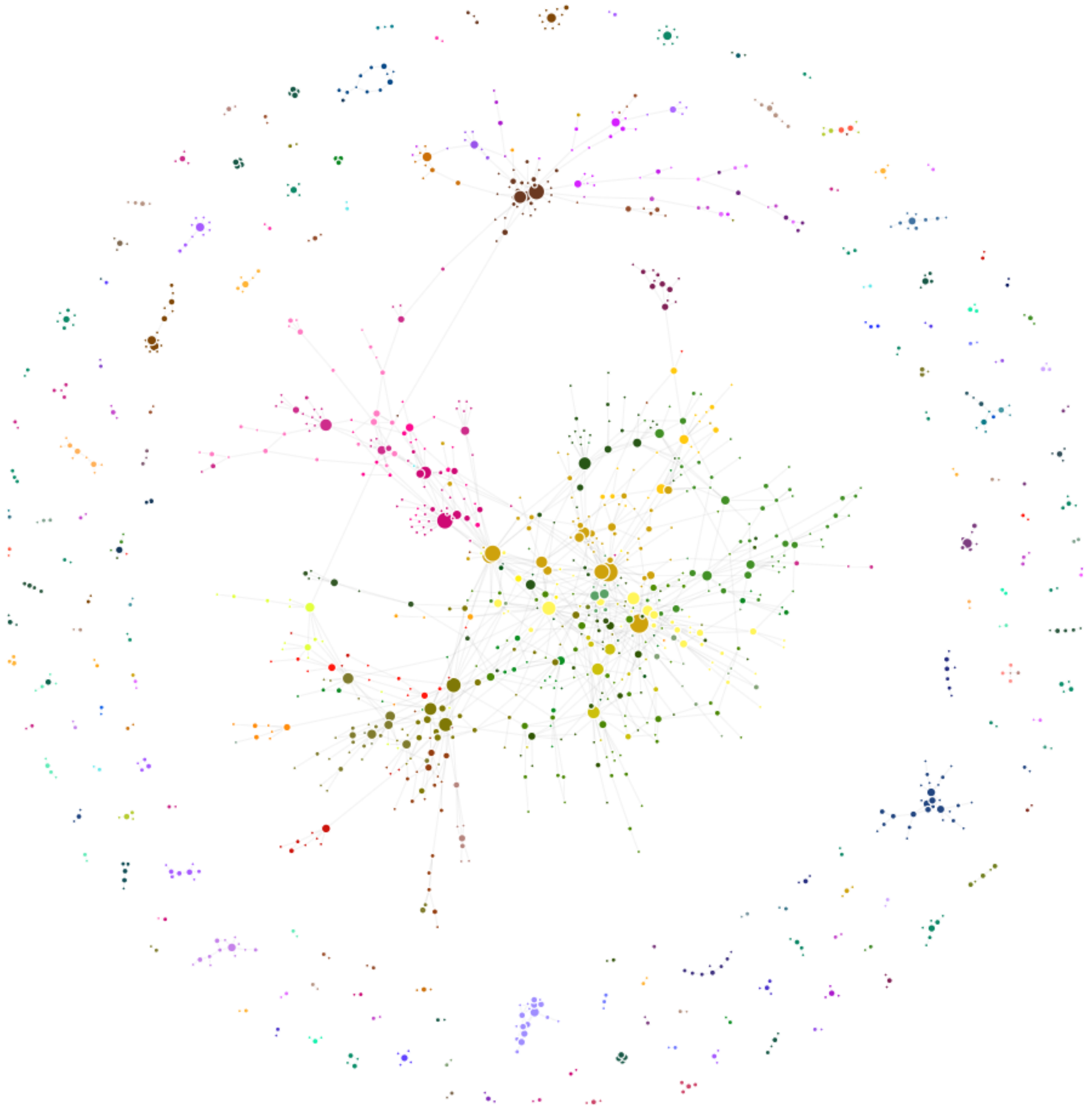
"Rule of 7"
Unknown Source
Presented by Cinda Heeren, 2016



Conflict-Free Final Exam Scheduling Graph

Unknown Source

Presented by Cinda Heeren, 2016

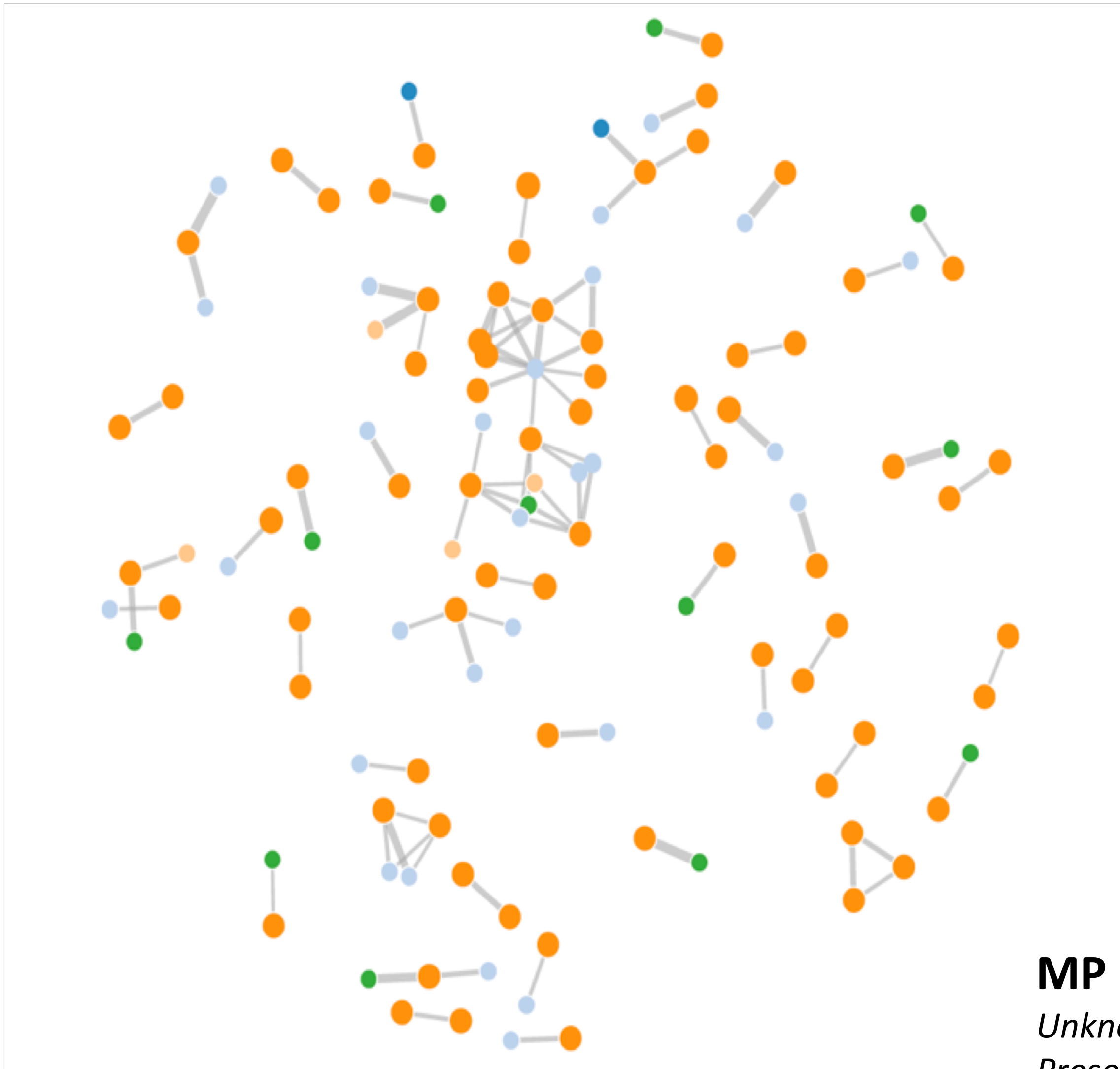


Class Hierarchy At University of Illinois Urbana-Champaign

A. Mori, W. Fagen-Ulmschneider, C. Heeren

Graph of every course at UIUC; nodes are courses, edges are prerequisites

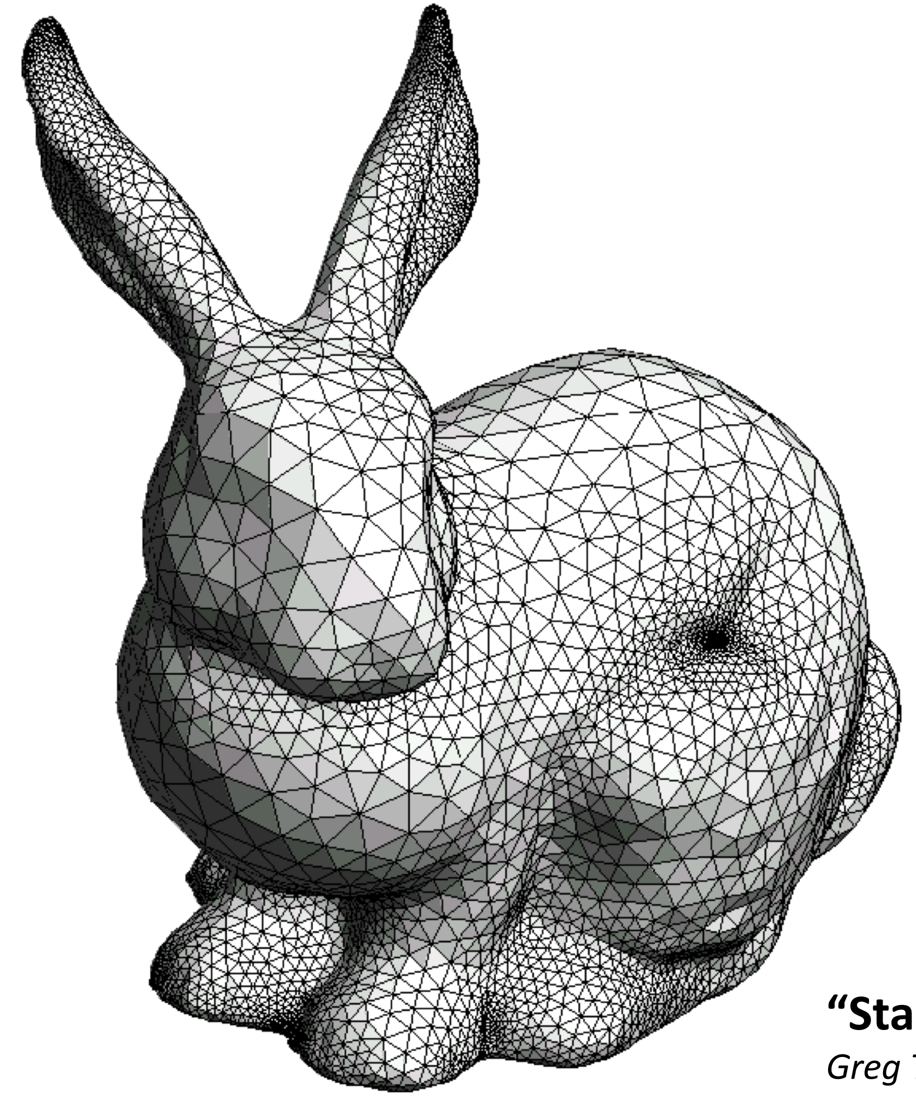
http://waf.cs.illinois.edu/discovery/class_hierarchy_at_illinois/



MP Collaborations in CS 225

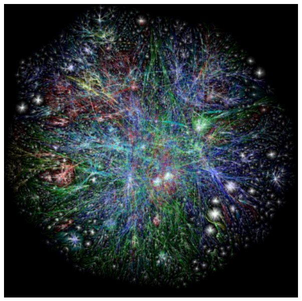
Unknown Source

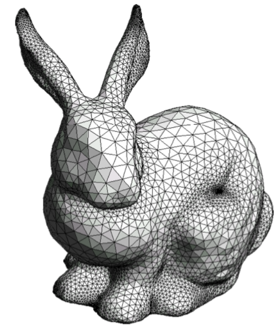
Presented by Cinda Heeren, 2016



“Stanford Bunny”
Greg Turk and Mark Levoy (1994)

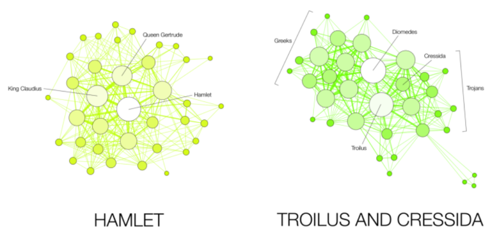
Graphs

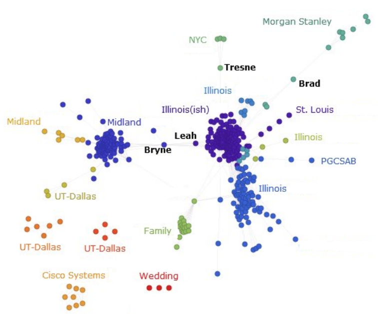


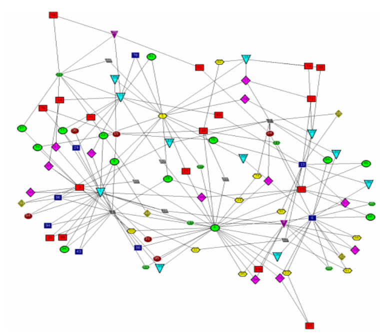


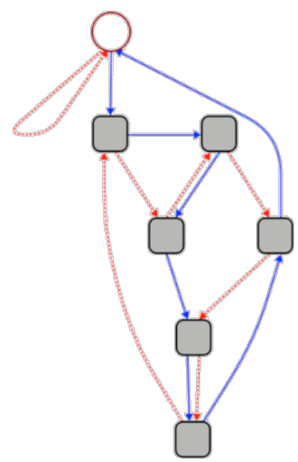
To study all of these structures:

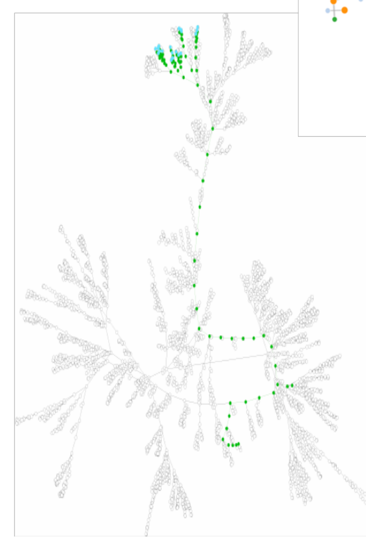
1. A common vocabulary
2. Graph implementations
3. Graph traversals
4. Graph algorithms

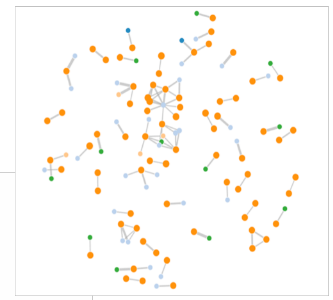












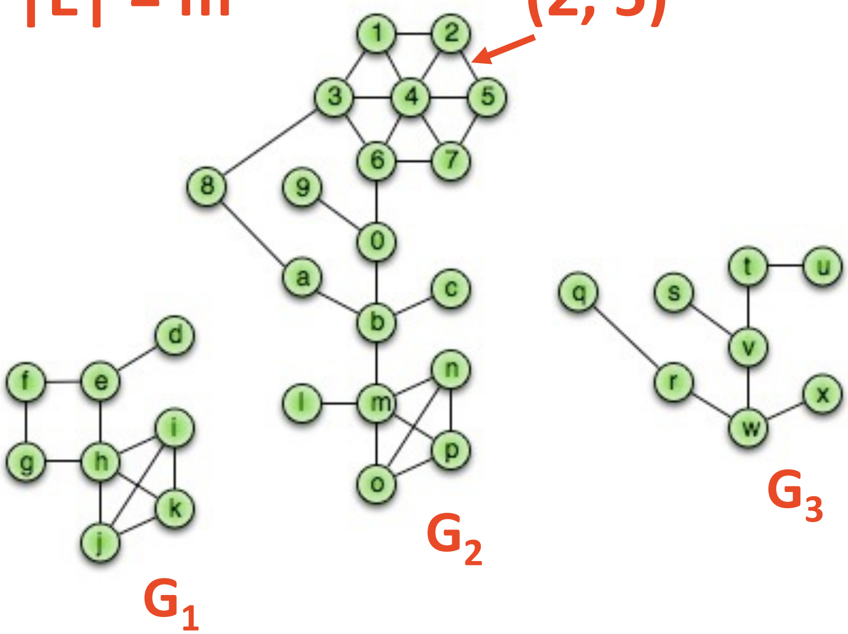
Graph Vocabulary

G = (V, E)

|V| = n

|E| = m

(2, 5)



Incident Edges:
I(v) = { (x, v) in E }

Degree(v): |I|

Adjacent Vertices:
A(v) = { x : (x, v) in E }

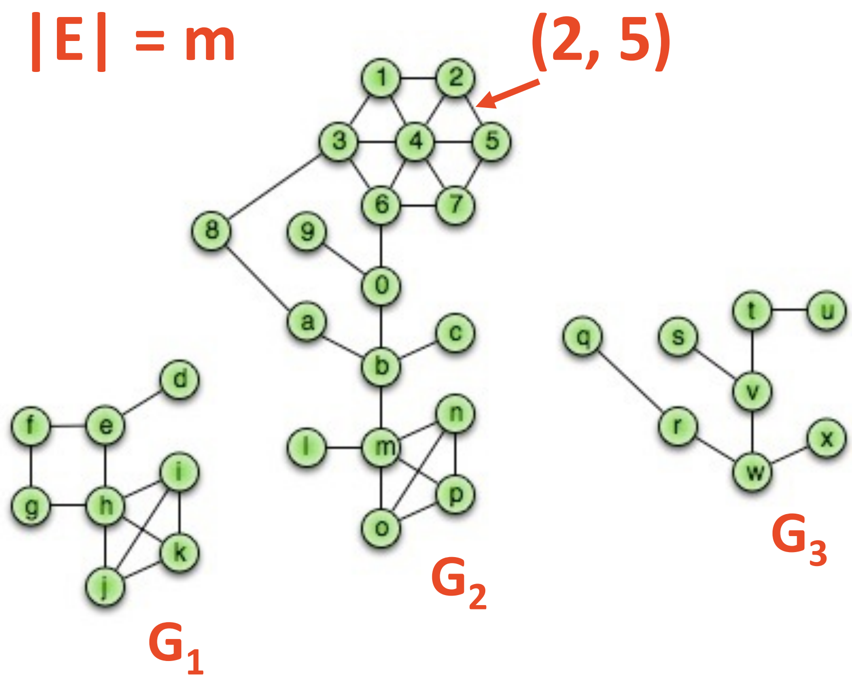
Path(G_2): Sequence of vertices connected by edges

Cycle(G_1): Path with a common begin and end vertex.

Simple Graph(G): A graph with no self loops or multi-edges.

Graph Vocabulary

$G = (V, E)$
$|V| = n$
$|E| = m$

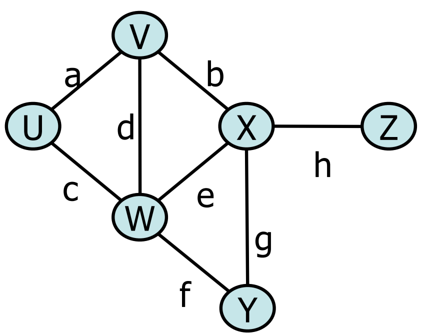


Subgraph(G):
$G' = (V', E')$:
$V' \in V$, $E' \in E$, and
$(u, v) \in E \rightarrow u \in V', v \in V'$

Complete subgraph(G)
Connected subgraph(G)
Connected component(G)
Acyclic subgraph(G)
Spanning tree(G)

Running times are often reported by **n**, the number of vertices, but often depend on **m**, the number of edges.

How many edges?



Minimum edges:

Not Connected:

Connected*:

Maximum edges:

Simple:

Not simple:

$$\sum_{v \in V} \deg(v) =$$

Connected Graphs