CS 225

#39: Prim's Algorithm

April 22, 2019 · *Fagen-Ulmschneider, Zilles*

A **Minimum Spanning Tree** is a spanning tree with the **minimal total edge weights** among all spanning trees.

- Every edge must have a weight
  - The weights are unconstrained, except they must be additive *(eg: can be negative, can be non-integers)*
- Output of a MST algorithm produces G':
  - G' is a spanning graph of G
  - G' is a tree

G' has a minimal total weight among all spanning trees. *There may be multiple minimum spanning trees, but they have equal total weight!*

- We covered the first classical algorithm (Kruskal) already!

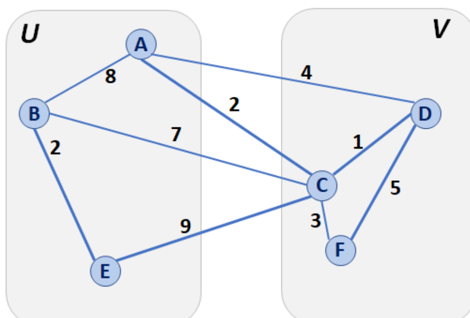
Partition Property

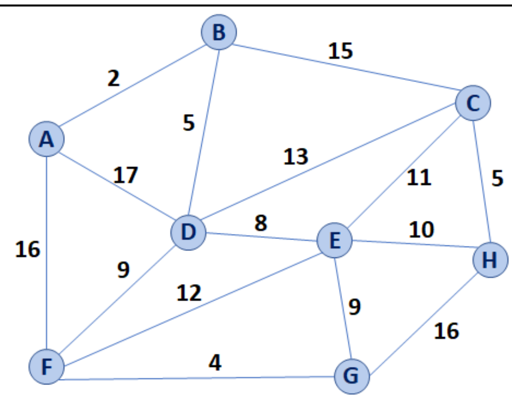
Consider an arbitrary partition of the vertices on **G** into two subsets **U** and **V**.

Let **e** be an edge of minimum weight across the partition.

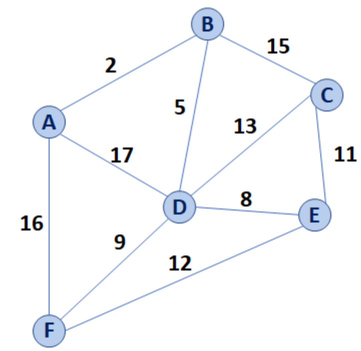
Then **e** is part of some minimum spanning tree.

Proof in CS 374!





Prim's MST Algorithm



Pseudocode for Prim's MST Algorithm

```
PrimMST(G, s):
  Input: G, Graph;
         s, vertex in G, starting vertex of algorithm
  Output: T, a minimum spanning tree (MST) of G

  foreach (Vertex v : G):
    d[v] = +inf
    p[v] = NULL
  d[s] = 0

  PriorityQueue Q   // min distance, defined by d[v]
  Q.buildHeap(G.vertices())
  Graph T           // "labeled set"

  repeat n times:
    Vertex m = Q.removeMin()
    T.add(m)
    foreach (Vertex v : neighbors of m not in T):
      if cost(v, m) < d[v]:
        d[v] = cost(v, m)
        p[v] = m

  return T
```

	Adj. Matrix	**Adj. List**
Heap		
Unsorted Array		

Running Time of MST Algorithms

Kruskal's MST	Prim's MST

Q: What must be true about the connectivity of a graph when running an MST algorithm?

…what does this imply about the relationship between **n** and **m**?

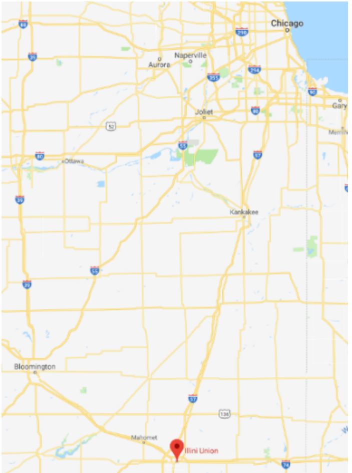
Kruskal's MST	Prim's MST

Q: Suppose we built a new heap that optimized the decrease-key operation, where decreasing the value of a key in a heap updates the heap in amortized constant time, or O(1)*. How does that change Prim's Algorithm runtime?

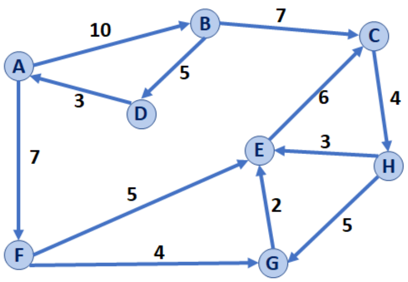
Final big-O Running Times of classical MST algorithms:

Kruskal's MST	Prim's MST

Shortest Path Home:



Dijkstra's Algorithm (Single Source Shortest Path)



Dijkstra's Algorithm Overview:

- The overall logic is the same as Prim's Algorithm
- We will modify the code in only two places – both involving the update to the distance metric.
- The result is a directed acyclic graph or DAG

CS 225 – Things To Be Doing:
1. **MP7 Live – Slightly different structure:**   **Hard Deadline** on Monday, April. 22 (TONIGHT) for Part 1   2. lab_finale in lab this week!   3. Daily POTDs are ongoing for +1 point /problem